Lahiri & Pal : Quantum Field Theory
1st edition
Answers to selected exercises

Ex. 1.3 •

$$\text{Hamiltonian :} \qquad H = \sum_{i=1}^{N} \Big(a_i^\dagger a_i + \frac{1}{2} \Big) \hbar\omega_i \,,$$

$$\text{state :} \qquad |n_1, n_2, \cdots, n_N\rangle = \prod_{i=1}^{N} \frac{\Big(a_i^\dagger\Big)^{n_i}}{\sqrt{n_i!}} \, |0\rangle \;.$$

$$\text{number operator :} \qquad \mathscr{N} = \sum_{i=1}^{N} a_i^\dagger a_i \,.$$

Ex. 1.7 • Maxwell equations are given in Ch. ??. The Lorentz force law on a particle with charge q is

$$\frac{dp^\mu}{d\tau} = qF^{\mu\nu} \frac{dx_\nu}{d\tau} \,.$$

where x^μ are the coordinates of the worldline of the particle, and τ is proper time, $d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$.

Ex. 1.8 • $\tau = 2 \times 10^{-6}$ s.

Ex. 2.2 • $(\Box + m^2)\phi = -\dfrac{\partial V}{\partial \phi}$.

Ex. 2.3 • $(\Box + m^2)\phi^\dagger = -\dfrac{\partial V}{\partial \phi}$.

Ex. 2.4 • The answer appears in Ch. 8.

Ex. 2.5 • $\Pi^i = \dot{A}^i$.

Ex. 2.6 • $\mathscr{H} = |\dot{\phi}|^2 + |\boldsymbol{\nabla}\phi|^2 + m^2 \phi^\dagger \phi + V(\phi^\dagger \phi)$.

Ex. 2.7 •

$$\begin{array}{ll} a) & p_k = l\dot{q}_k\,. \\ b) & H = \dfrac{1}{2l}\displaystyle\sum_{k=1}^{\infty}\Big(p_k^2 + l^2\omega_k^2 q_k^2\Big)\,. \\ c) & [a_k, a_m^\dagger]_- = \delta_{km}\,, \quad [a_k, a_m]_- = [a_k^\dagger, a_m^\dagger]_- = 0\,. \\ d) & H = \displaystyle\sum_{k=1}^{\infty}\dfrac{\hbar\omega_k}{2}(a_k a_k^\dagger + a_k^\dagger a_k)\,. \end{array}$$

Ex. 2.8 •

$$\begin{array}{ll} a) & j^\mu = iq(\phi^\dagger\partial^\mu\phi - \phi\partial^\mu\phi^\dagger)\,. \\ b) & j^\mu = iq(\phi^\dagger\partial^\mu\phi - \phi\partial^\mu\phi^\dagger + 2iqA^\mu\phi^\dagger\phi)\,. \end{array}$$

Ex. 2.9 • $j^\mu = \partial^\mu(x_\nu\phi)\partial^\nu\phi - x^\mu\mathscr{L}\,.$

Ex. 3.5 • $P^\mu = \frac{1}{2}\displaystyle\int d^3p\; p^\mu\Big(a^\dagger(p)a(p) + a(p)a^\dagger(p)\Big).$

Ex. 3.7 • $Q = iq\displaystyle\int d^3p\,\Big[a_1^\dagger(p)a_2(p) - a_2^\dagger(p)a_1(p)\Big].$

Ex. 4.7 • $m\overline{v}_r(\boldsymbol{p})\gamma^\mu v_s(\boldsymbol{p}) = -p^\mu\overline{v}_r(\boldsymbol{p})v_s(\boldsymbol{p})\,.$

Ex. 4.8 • $\overline{v}(\boldsymbol{p}')\,\gamma^\mu\,v(\boldsymbol{p}) = -\frac{1}{2m}\,\overline{v}(\boldsymbol{p}')\,[(p+p')^\mu - i\sigma^{\mu\nu}q_\nu]\,v(\boldsymbol{p})\,.$

Ex. 4.11 • Same as in Eqs. (4.60) and (4.61).

Ex. 4.16 • See errata.

Ex. 4.19 • See errata. $\mathscr{M}^\lambda{}_{\mu\nu} = \frac{1}{2}\overline{\psi}\gamma^\lambda\sigma_{\mu\nu}\psi - \Big(T^\lambda{}_\mu x_\nu - T^\lambda{}_\nu x_\mu\Big)$, where $T^\lambda{}_\mu = \overline{\psi}i\gamma^\lambda\partial_\mu\psi - g^\lambda{}_\mu\mathscr{L}\,.$

Ex. 6.2 • $\dfrac{G_\beta}{\sqrt{2}}\,\overline{\psi}_{(\nu_e)}\gamma^\mu(1-\gamma_5)\psi_{(e)}\;\overline{\psi}_{(n)}\gamma_\mu(1-\gamma_5)\psi_{(p)}\,.$

Ex. 6.4 •

$$\begin{aligned} S_{fi}^{(3c)} &= (-ih)^3\,(2\pi)^4\delta^4(k-p-p')\left[\frac{1}{\sqrt{2\omega_k V}}\frac{1}{\sqrt{2E_p V}}\frac{1}{\sqrt{2E_{p'}V}}\right] \\ &\qquad \times\int\frac{d^4q}{(2\pi)^4}\;i\Delta_F(q)\,[\overline{u}_s(\boldsymbol{p})iS_F(p-q)\;iS_F(p)v_{s'}(\boldsymbol{p}')]\;, \\ S_{fi}^{(3d)} &= (-ih)^3\,(2\pi)^4\delta^4(k-p-p')\left[\frac{1}{\sqrt{2\omega_k V}}\frac{1}{\sqrt{2E_p V}}\frac{1}{\sqrt{2E_{p'}V}}\right] \\ &\qquad \times\int\frac{d^4q}{(2\pi)^4}\;i\Delta_F(q)\,[\overline{u}_s(\boldsymbol{p})iS_F(p)\;iS_F(p-q)v_{s'}(\boldsymbol{p}')]\;. \end{aligned}$$

Ex. 7.3 • Yes.

Ex. 7.5 • $\Gamma = \dfrac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\dfrac{m_e^2}{m_\mu^2} + \cdots\right)$.

Ex. 7.6 • $\dfrac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \dfrac{m_e^2}{m_\mu^2}\left(\dfrac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$.

Ex. 7.9 • 11 GeV approximately.

Ex. 8.1 • See errata for Eq 8.21. $\partial_\mu F^{\mu\nu} + M^2 A^\nu = j^\nu$.

Ex. 8.2 • $\theta(x) = \theta_0(x) + \int d^4x'\ G_0(x - x')f(x')$, where G_0 is the massless scalar propagator (Green's function for the wave equation) and θ_0 satisfies the wave equation.

Ex. 8.5 • Physical states: $a_r^\dagger(k)\,|0\rangle$ for $r = 1, 2$.

Ex. 8.6 • $:P^\mu: = \int d^3k\ k^\mu \sum_r a_r^\dagger(k)a_r(k)$, where $k^0 = \omega_k$. This expression is valid on physical states selected by the Gupta-Bleuler condition, so that only the transverse modes contribute for each k^μ in the integral. (Physical quantities like Hamiltonian and momentum are independent of ξ when restricted to physical states.)

Ex. 11.3 • $eF(0)$.

Ex. 12.2 • Define $E_{\mu\alpha|\nu\beta} = g_{\mu\nu}g_{\alpha\beta} - g_{\mu\beta}g_{\nu\alpha}$. Then $\pi_{\mu\nu\lambda\rho}(k_1, k_2, k_3, k_4) = ak_1^\alpha k_2^\beta k_3^\gamma k_4^\delta\Big(E_{\mu\alpha|\nu\beta}E_{\lambda\gamma|\rho\delta} + E_{\mu\alpha|\lambda\gamma}E_{\nu\beta|\rho\delta} + E_{\mu\alpha|\rho\delta}E_{\nu\beta|\lambda\gamma}\Big) + bk_1^\alpha k_2^\beta k_3^\gamma k_4^\delta\Big(E_{\mu\alpha|\mu'\nu'}E_{\nu\beta}^{\nu'\lambda'}E_{\lambda\gamma|\lambda'\rho'}E_{\rho\delta}^{\rho'\mu'} + E_{\mu\alpha|\mu'\nu'}E_{\lambda\gamma}^{\nu'\lambda'}E_{\nu\beta|\lambda'\rho'}E_{\rho\delta}^{\rho'\mu'} + E_{\mu\alpha|\mu'\nu'}E_{\nu\beta}^{\nu'\lambda'}E_{\rho\delta|\lambda'\rho'}E_{\lambda\gamma}^{\rho'\mu'}\Big)$, where a and b are two form factors which depend on all possible Lorentz invariant combinations of the momenta.

Ex. 12.7 • 1/128 approximately.

Ex. 13.12 • $Q_a = \varepsilon_{abc}\int d^3x\ \dot{\phi}_b\phi_c$.

Ex. 15.12 • −17%.

Ex. 15.18 • +1.9%