For the spinors v_+ and v_-, we chose the subscript to match with the eigenvalue of the helicity operator $\Sigma_{\boldsymbol{p}}$ of Eq. (4.72), i.e.

$$\begin{aligned} \Sigma_{\boldsymbol{p}} v_+ &= v_+ \\ \Sigma_{\boldsymbol{p}} v_- &= -v_- \, . \end{aligned} \tag{H.1}$$

However, with this choice, charge conjugation acts as

$$\begin{aligned} (v_+)_C \equiv C\gamma_0^{\mathsf{T}} v_+^* &= -u_- \\ (v_-)_C \equiv C\gamma_0^{\mathsf{T}} v_-^* &= u_+ \, , \end{aligned} \tag{H.2}$$

which could cause confusion if we wish to associate v_+ and v_- with positive and negative helicity positrons (and u_+ and u_- with positive and negative helicity electrons), since charge conjugation should not change helicity.

Most other texts define v_+ and v_- such that they are charge conjugates of u_+ and u_- (for $u_\pm$ their definitions agree with ours). In that case v_+ has negative helicity and v_- has positive helicity, i.e., if we write $v'_\pm$ for the v-spinors in those texts,

$$\begin{aligned} \Sigma_{\boldsymbol{p}} v'_+ &= -v'_+ \\ \Sigma_{\boldsymbol{p}} v'_- &= v'_- \, . \end{aligned} \tag{H.3}$$

This definition of $v_\pm$ can be reached from our definition of $v_\pm$ by exchanging v_+ and v_-, with a relative minus sign, as mentioned in the errata webpage.

Is this difference 'just' a matter of convention? The answer is yes, *provided* we arrange the convention on the annihilation and creation operators for antifermion states of definite helicity, $\widehat{f}_\pm, \widehat{f}^\dagger_\pm$, (see Sec. 4.6) such that the positive helicity spinor (v_+ for us) comes with the negative helicity creation operator $\widehat{f}^\dagger_-$, while v_- comes with $\widehat{f}^\dagger_+$ in Eq. (4.92) and similarly in Eq. (4.93).

In the more popular convention, v_+ has negative helicity and v_- has positive helicity, and they come with $\widehat{f}^\dagger_+$ and $\widehat{f}^\dagger_-$, respectively in Eq. (4.92). Similarly in Eq. (4.93).[1]

Why is this just a convention? The explanation is delectably subtle — physical properties belong to particle states created by creation operators, not to spinors. To understand this, let us first work out the helicities of the antifermion states.

[1]One book whose convention is similar to ours is *Quantum Field Theory* by C. Itzykson and J. B. Zuber, McGraw-Hill, 1980.

Helicity is related to spin, which is part of the conserved charge $J_{\mu\nu}$ corresponding to Lorentz symmetry. Let us define the 3-vector $\boldsymbol{J} \equiv (J^{23}, J^{31}, J^{12})$, i.e., $J_i = \frac{1}{2}\epsilon_{ijk}J^{jk}$, and project it along $\boldsymbol{p}$,

$$\boldsymbol{J}_{\boldsymbol{p}} = \frac{\boldsymbol{J}\cdot\boldsymbol{p}}{\mathrm{p}}\,. \tag{H.4}$$

We need the effect of this on the single positron state

$$\left|e^+(\boldsymbol{p}, s)\right\rangle \equiv \widehat{f}_s^\dagger(\boldsymbol{p})\left|0\right\rangle\,. \tag{H.5}$$

Here the label s stands for $+$ or $-$.

For the operator $\boldsymbol{J}$,

$$J_i\left|e^+(\boldsymbol{p}, s)\right\rangle = J_i\widehat{f}_s^\dagger(\boldsymbol{p})\left|0\right\rangle = [J_i, \widehat{f}_s^\dagger(\boldsymbol{p})]_-\left|0\right\rangle\,, \tag{H.6}$$

since rotational invariance of the vacuum implies $J_i\left|0\right\rangle = 0$. The commutation relation between $\psi(x)$ and operator $J_{\mu\nu}$ is given in Eq. (4.89) and corrected in the erratum page as follows:

$$\Big[\psi(x), J_{\mu\nu}\Big]_- = i(x_\mu\partial_\nu - x_\nu\partial_\mu)\psi(x) + \frac{1}{2}\sigma_{\mu\nu}\psi(x)\,. \tag{H.7}$$

Using the plane wave expansion of Eq. (4.92) and the normalization relations of Eq. (4.48), we obtain

$$\widehat{f}_r^\dagger(\boldsymbol{k}) = \frac{1}{\sqrt{(2\pi)^3 2E_k}}\int d^3x\, e^{-ik\cdot x}v_r^\dagger(\boldsymbol{k})\psi(x)\,. \tag{H.8}$$

It follows that

$$[\boldsymbol{J}_{\boldsymbol{p}}, \widehat{f}_s^\dagger(\boldsymbol{p})]_- = -\frac{1}{2}v_s^\dagger(\boldsymbol{p})\Sigma_{\boldsymbol{p}}\int\frac{d^3x}{\sqrt{(2\pi)^3 2E_p}}e^{-ip\cdot x}\psi(x)\,, \tag{H.9}$$

so that, using Eq. (4.92), we get

$$\boldsymbol{J}_{\boldsymbol{p}}\widehat{f}_s^\dagger(\boldsymbol{p})\left|0\right\rangle = -\frac{1}{2}\frac{1}{2E_p}v_s^\dagger(\boldsymbol{p})\Sigma_{\boldsymbol{p}}v_r(\boldsymbol{p})\widehat{f}_r^\dagger(\boldsymbol{p})\left|0\right\rangle\,. \tag{H.10}$$

Remember that $\Sigma_{\boldsymbol{p}}v_\pm(\boldsymbol{p}) = \pm v_\pm(\boldsymbol{p})$ for our choice of the v spinors. If we now follow the modified convention mentioned earlier and write

$$v_r(\boldsymbol{p})\widehat{f}_r^\dagger(\boldsymbol{p}) \equiv v_-(\boldsymbol{p})\widehat{f}_+^\dagger(\boldsymbol{p}) - v_+(\boldsymbol{p})\widehat{f}_-^\dagger(\boldsymbol{p}) \tag{H.11}$$

(as opposed to the naive sum over r), we find

$$\boldsymbol{J_p} \left| e^+(\boldsymbol{p}, \pm) \right\rangle \equiv \boldsymbol{J_p} \widehat{f}^\dagger_\pm(\boldsymbol{p}) \left| 0 \right\rangle = \pm \frac{1}{2} \left| e^+(\boldsymbol{p}, \pm) \right\rangle \,, \qquad \text{(H.12)}$$

which is exactly what we should expect.

For fermions, the corresponding equation can be worked out by taking the adjoint of Eq. (H.7), and we find

$$\boldsymbol{J_p} \left| e^-(\boldsymbol{p}, \pm) \right\rangle \equiv \boldsymbol{J_p} f^\dagger_\pm(\boldsymbol{p}) \left| 0 \right\rangle = \pm \frac{1}{2} \left| e^-(\boldsymbol{p}, \pm) \right\rangle \,, \qquad \text{(H.13)}$$

where now $f^\dagger_s(\boldsymbol{p})\overline{u}_s(\boldsymbol{p}) \equiv f^\dagger_+(\boldsymbol{p})\overline{u}_+(\boldsymbol{p}) + f^\dagger_-(\boldsymbol{p})\overline{u}_-(\boldsymbol{p})$.

The relative sign in $v_r(\boldsymbol{p})\widehat{f}^\dagger_r(\boldsymbol{p})$ is needed to make the fermion states charge conjugates of the antifermion states with same helicity and without a relative phase. That is, if charge conjugation acts on the *spinors* in the Dirac field as $\psi_C \equiv \mathcal{C}\psi\mathcal{C}^{-1} = \mathsf{C}\gamma_0^{\mathsf{T}}\psi^*$ (see Sec. 10.3), we find using Eq. (H.2)

$$\begin{aligned} \mathcal{C} f^\dagger_+ \mathcal{C}^{-1} &= \widehat{f}^\dagger_+ \,, \qquad & \mathcal{C} f^\dagger_- \mathcal{C}^{-1} &= \widehat{f}^\dagger_- \,, \\ \mathcal{C} \widehat{f}^\dagger_+ \mathcal{C}^{-1} &= f^\dagger_+ ,, \qquad & \mathcal{C} \widehat{f}^\dagger_- \mathcal{C}^{-1} &= f^\dagger_- \,. \end{aligned} \qquad \text{(H.14)}$$

So charge conjugation takes fermion states to antifermion states and vice versa, *without* any effect on their spins.

Why was there a clash of conventions? The reason is that the spinors u and v are plane wave soutions of Dirac equation, but they *cannot* represent wave functions of single particles, because that leads to negative energy states. Physical properties like helicity and charge (conjugation) should not be attributed to spinors, but only to particle states constructed with creation operators. But spinors multiply creation and annihilation operators in the plane wave expansion of the field. As long as we choose the correct spinor as the coefficient of the correct operator, our results will be correct.

Finally, conventions are meant to simplify and assist calculations. If you find our convention confusing, and in particular find Eq. (H.11) unsatisfactory, you are welcome to use the other convention, but then you will have to live with Eq. (H.3). In the book we have not calculated any amplitude involving initial or final antifermion states of definite helicity, so there should be no confusion while reading the book.

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